cases studied here, but a large amount of computation would be required in order to determine the best functional representation for every structure. However, it appears that the proposals put forward here could lead to a new form of shielded suspended substrate microstrip line. For situations where several circuits are etched upon the one substrate, high-permittivity substrate materials are required to isolate these circuits, and the multilayer structure proposed provides a means of reducing the losses introduced by such materials. Further work is in hand to define the effects on the electric field distribution of moving the substrate and to find an optimum structure.

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Waves Guided Between Open Parallel Concave Reflectors

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Abstract—**Wave propagation between two open parallel concave reflectors is usually considered by a wave-beam or a multiple-reflection approach. It is shown that the field distribution in elliptic waveguides, for specific wave modes, approaches that in open reflector waveguides.**

INTRODUCTION

A structure composed of two parallel cylindrical reflectors, as illustrated in Fig. 1, has attractive characteristics as a waveguide, particularly in the millimeter-wave region. Three methods for the analysis of such reflector waveguides are described in the literature. One is based on a description of the field distribution by transverse wave beams [1]–[3]; another one [4], [5] uses Huygen's principle applied to the iterative radiation from and reflection by the reflector surfaces. The third method [6], [7] analyzes the case as a boundary value problem by taking into consideration diffraction at the openings. In this analysis, the wave equation is being transformed into a parabolic partial differential equation. The use of these reported methods involves restrictions such as the limitation to confocal reflectors [1]–[6] and restrictions to specific ratios of cross-sectional dimensions associated with the choice of elliptic coordinates [6].

In the present short paper, it is shown that the wave propagation in open waveguides with concave reflectors also can be described by that in a waveguide with elliptic cross section. The analysis indicates that for large and transverse wavenumbers the field distribution in the closed elliptical waveguide approaches that of the open reflector-type waveguide. In contrast to the previously reported approaches, the new method does not require implementation of any of the above-mentioned restrictions.[1]

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[1] A similar approach has been followed by Toraldo di Francia [11] in the analysis of a "flat-roof" resonator. This has been brought to the attention of the authors by one of the reviewers.

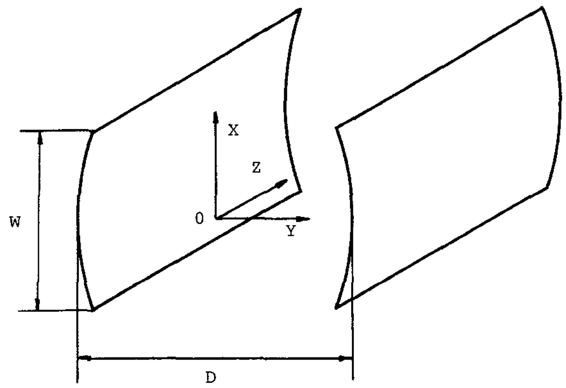

Fig. 1. Reflector guide.

BASIC RELATIONSHIPS

In this section the basic relationships for the field distribution in the elliptic waveguide are outlined. For large values of the parameter q, the equations then also describe the field distributions in the open waveguide with concave reflectors.

The transmission characteristics of elliptic waveguides have been studied by several investigators [8], [9]. An elliptic coordinate system is usually used in the analysis, as indicated in Fig. 2. The distributions of the various field components are found by solving the transverse wave equation for the longitudinal field components E_z and H_z in this coordinate system. Assuming harmonic time variations and wave propagation in the positive z direction (along the cylinder axis), the wave equations have the basic form

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\eta^2} + 2q(\cosh 2\xi - \cos 2\eta)\psi = 0 \tag{1}$$

where $\psi = E_z$ for TM modes and $\psi = H_z$ for TE modes. Other parameters are

$q \quad = k_c^2c^2/4$;
$k_c^2 \quad = \omega^2\epsilon_0\mu_0 - k_z^2$;
$2c$ focal distance;
k_z propagation constant in z direction;
k_c propagation constant in transverse direction (also propagation constant for cutoff).

The basic solution of (1) is found by separation of variables and becomes, in customary writing,

$$\psi_{m,n} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{m,n}Ce_m(q_{m,n}\xi)ce_m(q_{m,n}\eta) \tag{2a}$$

for even-field modes, and

$$\psi_{m,n} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{m,n}Se_m(q_{m,n}\xi)se_m(q_{m,n}\eta) \tag{2b}$$

for odd-field modes.

The functions ce_m and se_m are the even and odd Mathieu functions of order m, and Ce_m and Se_m are the corresponding modified Mathieu functions. The order number m indicates the number of zeros of the Mathieu functions between $\eta = 0$ and $n = \pi/2$. In the present case, the functions with the order numbers m and n then represent the distributions of E_z and H_z, respectively, for the various wave modes. Evaluation of the Mathieu functions shows that for large values of q and k_c the functions Se_m and Ce_m are highly quasi-periodic. The number of zeros increases with increasing values of q. The functions, in the present case, represent the standing waves resulting from reflections between the upper and lower parts of the elliptic waveguide, shown in Fig. 2, along the Y axis. Considering the field distribution in direction of η (X direction), it is described by ce_m and se_m. For the fundamental mode designated by $m = 0$, the function ce_0 has no periodicities and zeros, and it increases monotonically when η varies between 0 and $\pi/2$. Typical examples of these functions are illustrated in Fig. 3 for various values of the parameter q. The curves indicate that for large values of q the magnitude of ce_0 is large in the vicinity of $\eta = \pi/2$ and becomes negligibly small near $\eta = 0$. This means that the field distributions

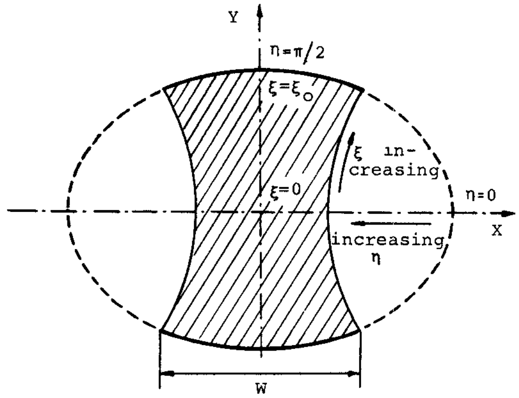

Fig. 2. Elliptic coordinate system and reflector guide as a cut section of closed elliptic waveguide.

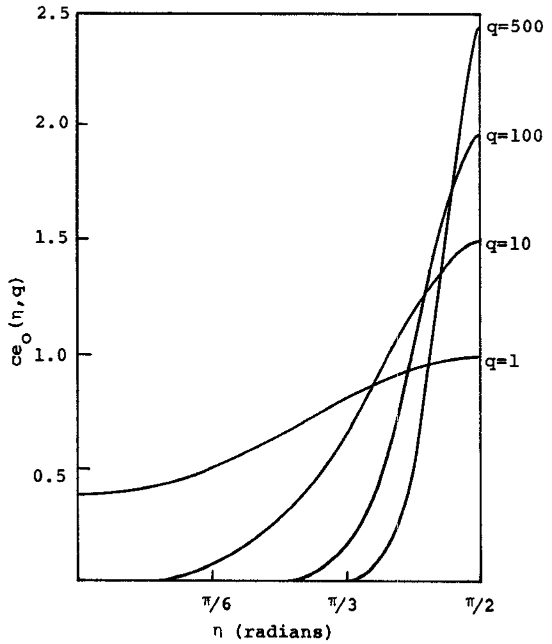

Fig. 3. Mathieu function ce_0 (η, q) versus angular coordinate η for various values of q (field distribution in X direction).

become concentrated in the region close to the Y axis $(\eta = \pi/2)$ in the waveguide (Fig. 2). The degree of concentration increases with increasing value of q and, correspondingly, the magnitude of the electromagnetic field decreases rapidly with increasing distance from the Y axis. Hence, for large values of q, portions of the conducting wall of the elliptic waveguide can be removed in the regions where the field magnitude is negligible. This is indicated in Fig. 2 by the dashed parts of the elliptic boundary. What then remains is an open waveguide of width W with a cross section similar to that shown in Fig. 1 being turned around 90°. The field distribution in elliptic waveguides thus can be well used for the description of the fields in open waveguides for large values of q encountered for oversized dimensions of the distance D between the two cylindrical reflectors $(D \gg \lambda)$. This requirement is always satisfied in practical open reflector-type waveguides.

COMPARISON OF THE RESULTS WITH THOSE OBTAINED BY OTHER APPROACHES

It was shown by Morse and Feshback [10] that for large values of the parameter q (large number of periods of the standing waves between the reflectors) the Mathieu functions ce_m and se_m can be approximated by series of Gaussian–Hermite functions as

$$ce_m(q,\eta) = A_m \sum_{\nu=-\infty}^{\infty} [\exp(-z_1^2/2)H_m(z_1) + \exp(-z_2^2/2)H_m(z_2)],$$

$$se_m(q,\eta) = B_m \sum_{\nu=-\infty}^{\infty} [\exp(-z_1^2/2)H_{m-1}(z_1) - \exp(-z_2^2/2)H_{m-1}(z_2)].$$

In these equations, we have

$$z_1 = (4q)^{1/4}(\eta - \pi/2 + 2\nu\pi)$$
$$z_2 = (4q)^{1/4}(\eta + \pi/2 + 2\nu\pi).$$

We observe that these functions for $m = 0$ are similar to those shown in Fig. 3. The series of Gaussian–Hermite functions, representing as an approximation the Mathieu functions, are identical to the functions found by considering wave propagation in the reflector guide based on the transverse wave-beam approach according to Schwering [1] and Nakahara [2].

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Surface Wave on a Ferrite Magnetized Perpendicular to the Interface

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***Abstract*—A semi-infinite ferrite magnetized perpendicular to the interface is found to support a nonmagnetostatic surface wave. The wave exists within a finite wavelength band and a finite frequency band. Since the wavelength is typically of the order of 1 cm, large samples of polycrystalline ferrite material may be used for the experiments.**

The propagation of magnetostatic surface waves on a ferrite slab magnetized in the plane of the slab and transversely to the direction of propagation has been studied theoretically [1] as well as experimentally [2], [3]. The effects on the magnetostatic surface wave produced by adding a conductive plate or a dielectric layer backed by a conductor to one side of the ferrite slab have also been investigated [4]–[6]. Morgenthaler [7] and Jao[8] have studied the propagation of magnetostatic surface waves on a ferrite slab magnetized transversely to the direction of propagation, but at an arbitrary angle θ to the ferrite slab surface. They found that for θ in the range $\theta_c < \theta < \pi - \theta_c$, where θ_c $(\theta_c < \pi/2)$ is the critical angle, no magnetostatic surface wave exists. Nonmagnetostatic surface waves termed "dynamic mode" or "dielectrically induced" surface waves have been studied theoretically for a semi-infinite ferrite magnetized parallel to the interface and transversely to the direction of propaga-

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